Gravitational Waves

Wiggles in the fabric of space-time

Background: Einstein's General Theory of Relativity

- Principle of Equivalence
- Gravity due to nature of space-time
- Gravitational field is metric tensor
- Attractive nature of gravity
  - Negative mass not allowed
  - Exchange of spin-2 quanta attractive
- Acceleration of mass (quadrupole or higher) produces gravity waves

Gravitational Wave Energy

- Start with Einstein’s GR: Space-time warp by local masses
- Find expression for gravitational energy in space-time near a local distribution of mass
- Assume far from local region to get radiation field
- Radiation field has two components with tensor polarization

Einstein's formula

$$\frac{dE}{dt} = -\frac{G}{5c^5}\dddot{Q}\cdot\dddot{Q}$$

$$Q_{ij} = \int \rho \left(x_i x_j - \frac{1}{3} r^2 \delta_{ij} \right) dV$$

Application to Binary Pulsars

- Very accurate orbital period from pulse time
- Very low energy loss
- Keplerian orbits (if speed << c)
- Find average power loss over full period

Result

$$\frac{dE}{dt} = -\frac{2}{5(m_1+m_2)c^5}\left(\frac{m_1 m_2}{m_1+m_2}\right)^2\left(2^{22}\pi^{10}\frac{(GM)^7}{T^{10}}\right)^{1/3}\frac{\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)}{(1-e^2)^3}$$